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**ON THE BED EXPANSION IN AGGREGATIVE FLUIDIZATION**

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Predictions of two hydrodynamic models and four entirely empirical correlations for the expansion of freely bubbling fluidized beds are compared with four sets of the experimental data. The results provide answers to some practical questions regarding the applicability of the recently proposed relationships for predicting the expansion in the fluidized beds with larger particles.

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The expansion characteristics is a basic parameter associated with the fluidized bed behaviour. Information on this property is essential for establishing the efficiency of contact between the gas and solid. The overall bed expansion and mean bed voidage are important parameters in the design, modelling and control of fluidized bed reactors. Under given conditions of operations, these parameters determine the mean residence time of gas in the fluidized bed.

While liquid–solid systems usually expand homogeneously (particulate fluidization, Hartman et al.<sup>1</sup>), gas–solid systems are generally associated with the formation and flow of nonhomogenities (bubbles) through the fluidized beds (aggregative fluidization).

The bed expansion is closely related to the bubbling phenomena in the aggregative fluidized beds. It means that physical considerations on the bed expansion include aspects such as bubble formation at a given distributor, bubble coalescence and splitting<sup>2</sup>, bubble frequency, bubble growth<sup>3–6</sup>, possible slugging<sup>7</sup>, size distribution of bubbles<sup>8</sup>, rise velocity of bubbles<sup>9</sup>, division of the ascending gas between the bubble and emulsion phase<sup>10</sup>. In general, these phenomena are or can be dependent upon a number of different factors such as average size, density and shape of particles, particle size distribution, gas density and viscosity, bed geometry, presence of internals in the bed, operating temperature and pressure, interparticle forces and electrostatic effects. All the above circumstances contribute, particularly in their combination, to the complexity of the bubbling phenomena, which are still far from being fully understood.

This brief communication is a sequel to a previous work of ours<sup>11</sup> on the expansion of the bed of larger particles belonging to Group B or D of Geldart's classifica-

tion<sup>12</sup>. The purpose of this brief study is to explore the predictions of bed expansion based on the two-phase model combined with the theory of bubble growth and those provided by very simple empirical correlations available in the literature.

The mean voidage of bed is usually determined from the measured height of an expanded bed according to the relationship

$$\bar{\varepsilon} = 1 - W/(FH\rho_s). \quad (1)$$

The local voidage can be obtained from measurements of the axial pressure gradient by means of

$$\varepsilon = 1 - \frac{1}{(\rho_s - \rho_g)g} \left( \frac{dP}{dh} \right). \quad (2)$$

In some studies, X-ray and  $\gamma$ -ray absorption and capacitance probe were employed in the experimental work.

The bed expansion occurs in a fluidized bed of larger particles primarily as a result of the rise and growth of bubbles. As the void fraction of the dense (interstitial, emulsion) phase is commonly assumed to remain constant at  $\varepsilon_{mf}$ , the overall fraction of bed occupied by bubbles,  $\bar{\varepsilon}_b$ , can be related to the bed voidage

$$\bar{\varepsilon}_b = (\bar{\varepsilon} - \varepsilon_{mf})/(1 - \varepsilon_{mf}). \quad (3)$$

On substituting

$$\bar{\varepsilon}_b = (H - H_{mf})/H \quad (4)$$

we can write for the bed expansion

$$H/H_{mf} = (1 - \varepsilon_{mf})/(1 - \bar{\varepsilon}). \quad (5)$$

For a freely bubbling bed we can express the fraction of the bed volume occupied by bubbles at the level  $h$  as

$$\varepsilon_b = \psi(U - U_{mf})/u_b, \quad (6)$$

where  $\psi$  denotes the ratio of the actual visible bubble flow rate to the gas flowrate given by  $(U - U_{mf})F$  and  $u_b$  is the mean, absolute rising velocity of bubbles. The experimental experience suggests that  $\psi$  is often smaller than unity and can be a function of the distance above the distributor. The simple two-phase models assume, however, that all gas in excess of that required for incipient fluidization passes through the bed as a visible bubble flow, i.e.  $\psi = 1$ .

Upon integrating Eq. (6) and substitution from Eq. (4) we can write for the height of the expanded bed

$$H = H_{mf} + (U - U_{mf}) \int_0^H \frac{\psi}{u_b} dh. \quad (7)$$

On the basis of experimental observations, Hillgardt and Werther<sup>13</sup> have modified the frequently employed relationship of Davidson and Harrison<sup>9</sup> for the bubble rise velocity as follows

$$u_b = \psi(U - U_{mf}) + 0.71\delta(gd_v)^{0.5}, \quad (8)$$

where

$$d_v = d_{v0}(1 + 27(U - U_{mf}))^{1/3} \cdot (1 + 6.84h)^{1.2}. \quad (9)$$

While the parameter  $\psi$  describes the deviation of the visible bubble flow from the simple two-phase theory, the parameter  $\delta$  accounts for the formation of bubble paths that are typical of fluidized bed with larger diameters (e.g., "gulf-streaming"). In general, both parameters depend on the bed dimensions and physical properties of the solids. For the sand particles from Geldart's Group B ( $d_p = 0.48$  mm,  $\rho_s = 2640$  kg m<sup>-3</sup>,  $U_{mf} = 0.18$  m s<sup>-1</sup>) fluidized with air at ambient conditions, the authors<sup>13</sup> described the hydrodynamic parameters  $d_{v0}$ ,  $\psi$  and  $\delta$  as follows

$$\begin{aligned} d_{v0} &= 0.0123 \text{ m} \\ \psi &= 0.26 \quad \text{for } h/D < 0.55 \\ \psi &= 0.35 (h/D)^{0.5} \quad \text{for } 0.55 \leq h/D \leq 8 \\ \delta &= 0.87 \quad \text{for } 0.1 \leq D \leq 1 \text{ m.} \end{aligned} \quad (10)$$

The implicit equation (7) can numerically be solved without any difficulty. We applied the similar approach in our preceding work<sup>11</sup>. Assuming  $\psi = \text{const.}$  and  $\delta = 1$ , we were able to integrate analytically Eq. (7) in which the bubble size was expressed by the commonly used correlation of Mori and Wen<sup>3</sup> and obtained the following relationship:

$$\begin{aligned} H = H_{mf} + \frac{\psi}{m} &\left[ \frac{1}{A^{0.5} - 1} \ln \frac{A^{0.5} + (A - B \exp(-mH))^{0.5}}{A^{0.5} + (A - B)^{0.5}} - \right. \\ &- \frac{1}{A^{0.5} + 1} \ln \frac{A^{0.5} - (A - B \exp(-mH))^{0.5}}{A^{0.5} - (A - B)^{0.5}} + \\ &\left. + \frac{2}{1 - A} \ln \frac{1 + (A - B \exp(-mH))^{0.5}}{1 + (A - B)^{0.5}} \right], \end{aligned} \quad (11)$$

$$\psi = \text{const.}, \delta = 1.$$

There are not many experimental data on the bed expansion available in the literature. The average height of a fluidized bed is difficult to determine, because the bed surface is subject to large fluctuations of a variable amplitude. The foregoing analysis suggests that the bed expansion can depend on the fluidizing gas velocity, height of the bed at quiescent conditions, particle size and density, diameter of the containing vessel, gas density and viscosity, design of the gas distributor and the presence of internals in the bed. This study centres on the freely bubbling beds. For the beds with internals, the reader is referred to other works<sup>14-16</sup>.

The pronounced increase in the average height of a fluidized bed with the excess gas velocity is nearly linear in the bubbly flow regime<sup>17-18</sup>. Unfortunately, the present state of the knowledge does not make it possible to specify unequivocally influence of other factors mentioned in the preceding paragraphs. This can be hidden within the considerable scatter of the experimental results.

Several entirely empirical correlations have been proposed for predicting the bed expansion of larger particles. These correlations fall into two main groups. In the first group, the correlations include the mean void fraction  $\bar{\varepsilon}$ . In the second group, the bed expansion occurs in terms of the expansion ratio  $H/H_{mf}$ .

Hsiung and Thodos<sup>19</sup> have employed a modified form of the Richardson-Zaki equation<sup>20</sup> which was originally proposed for particulate, liquid-solid, fluidization

$$\varepsilon = \varepsilon_{mf} \left( \frac{Re - k}{Re_{mf} - k} \right)^n, \quad (12)$$

where

$$k = 0.216Re_{mf}^{1.2} - 0.35 \quad (13)$$

and  $n = 0.28$  for the particle density  $\rho_s = 1\,040 \text{ kg m}^{-3}$ . As can be seen, Eq. (12) is normalized with respect to the reference state of minimum fluidizing conditions ( $Re_{mf}, \varepsilon_{mf}$ ). The alternative form of Eq. (12) is normalized by the authors<sup>19</sup> to the terminal (free fall) conditions ( $Re_t, 1$ ). Doichev and Boichev<sup>21</sup> have used a similar version of the Richardson-Zaki formula.

A correlation of the bed expansion based on the dimensionless analysis has been developed by Thonglimp et al.<sup>22</sup>

$$\varepsilon = 1.57Re^{0.29}/Ar^{0.19} \quad (14)$$

for  $d_p = 0.18 - 2.1 \text{ mm}$  and  $\rho_s = 1\,600 - 7\,400 \text{ kg m}^{-3}$ .

Tamarin and Teplickii<sup>23</sup> have fitted the experimental data amassed with different solids and columns by the following, very simple expression:

$$H/H_{mf} = 1 + 0.327H_{mf}(U - U_{mf})^{0.667}/D^{0.5} \quad (15)$$

for  $0.5 < H_{mf}/D < 2$ .

Babu et al.<sup>24</sup> have also presented a simple equation, which incorporates the effect of some physical properties of the system

$$\frac{H}{H_{mf}} = 1 + 14.3 \frac{d_p^{1.01} \rho_s^{0.376} (U - U_{mf})^{0.737}}{U_{mf}^{0.937} \rho_g^{0.126}} \quad (16)$$

It is of interest to note that the power at the excess gas velocity is close to a value of 0.7 in both correlations (15) and (16).

### EXPERIMENTAL

Measurements of bed expansion have been conducted in a 0.14 m i.d. glass column fitted with a perforated plate distributor of fluidizing air. The distributor of free area  $\phi = 6.9\%$  contained 151 orifices with a diameter of 0.2 cm. The flow rate of dried air was measured by a calibrated rotameter.

Ceramsite particles of mean diameter 1.0 mm (0.9–1.1 mm) were used in the experiments. This material is made by mechanical treatment and subsequent calcination of the claystone cover from lignite mines. The bulk density of the calcined ceramsite particles amounted to 1 600 kg m<sup>-3</sup>. The minimum fluidization velocity measured by the standard procedure was as high as 0.258 m s<sup>-1</sup> at ambient temperature and pressure.

### RESULTS AND DISCUSSION

The bed height was determined by direct visual observations as a mean level of the fluctuating surface of bed. At a given gas flow rate, the observations were generally repeated several times by two observers and the average value was determined. Such values of the measured bed heights are plotted in Fig. 1 against the superficial excess velocity of gas. As can be seen in this figure, curve 1 predicted by Eq. (11) is quite close to the experimental data points and follows their trend very well. The differences in bed expansions predicted by different equations range from 15 to 20% which is a reasonable agreement. Nevertheless, the appreciable difference, particularly between curves 1 and 4 is worth mentioning. Computations of the bubble size by Eq. (9) and from the correlation of Mori and Wen show that the predicted values are not much different. Since the hydrodynamic parameter  $\delta$  in Eq. (8) is close to unity, the low values of  $H/H_{mf}$  represented by curve 4 in Fig. 1 stem from small values of the parameter  $\psi$ . In contrast to this, our results with the ceramsite particles suggest that practically the entire gas flow in excess of the minimum fluidization velocity takes the form of bubbles, i.e.  $\psi \approx 1$ . The limited scope of the experiments does not make it possible to explore further this discrepant point. Apart from a smaller size of the experimental column, it should be mentioned that the ceramsite particles used in the experiments are sharp edged and irregular in their shape.

In addition to our results, experimental data available in the literature<sup>13,18,25</sup>

have also been employed for testing the expansion predictions. The basic physical characteristics of the experimental systems are presented in Table I.

As can be presumed and seen in Fig. 2, the correlations of Hillgardt and Werther<sup>13</sup> fit their own data with very good accuracy. The predictions of the equations of Tamarin and Teplickii<sup>23</sup> and those of Babu et al.<sup>24</sup> are 5 and 15%, respectively, above the experimental curve. As shown in Fig. 3, these two entirely empirical correlations fit well the experimental data of Best and Yates<sup>18</sup>. Comparison of the

TABLE I

Experimental data on fluidized bed expansion

Reference	Bed diameter m	Particle properties		
		material	$\rho_s$ $\text{kg m}^{-3}$	$\bar{d}_p$ mm
Hillgardt and Werther <sup>13</sup>	0.5	sand	2 640	0.48
Best and Yates <sup>18</sup>	0.1	alumina spheres	1 270	0.81
Denloye <sup>25</sup>	0.11 0.3 × 0.3	sand	2 600	1.02 2.6
This work	0.14	ceramsite	1 600	1.0

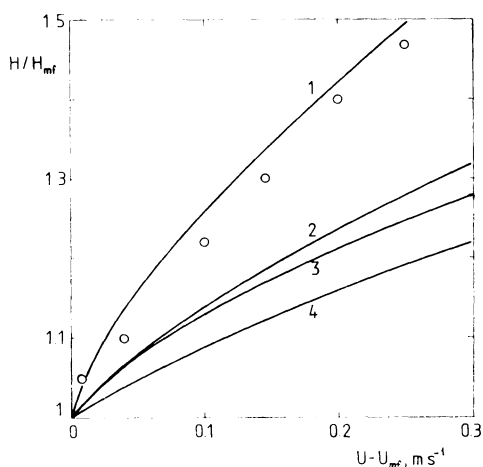


FIG. 1

Comparison of the bed expansion measured on the ceramsite particles-air system with the predictions of the different expressions. Experimental data points ( $\circ$ ),  $D = 0.14$  m,  $H_{mf} = 0.13$  m,  $d_p = 1$  mm, this work. The solid lines show the bed expansion ratio predicted by different equations. 1 Hartman et al.<sup>11</sup> for  $\psi = 1$ ,  $\delta = 1$ ; 2 Babu et al.<sup>24</sup>; 3 Tamarin and Teplickii<sup>23</sup>; 4 Hillgardt and Werther<sup>13</sup>

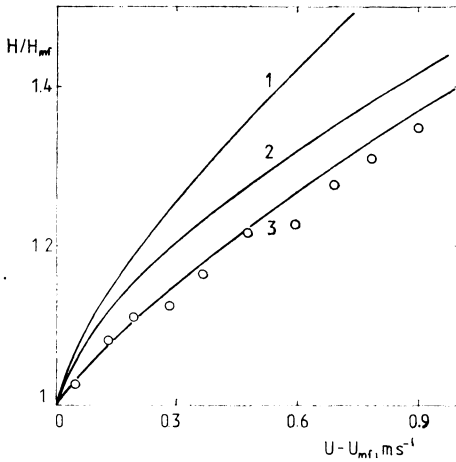


FIG. 2

Comparison of the bed expansion measured on the sand particles-air system with the predictions of different expressions. Experimental data points ( $\circ$ ),  $D = 0.5$  m,  $H_{mf} = 0.85$ ,  $\bar{d}_p = 0.48$  mm, Hilligardt and Werther<sup>13</sup>. The solid lines show the bed expansion ratio predicted by different equations. 1 Babu et al.<sup>24</sup>; 2 Tamarin and Teplickii<sup>23</sup>; 3 Hilligardt and Werther<sup>13</sup>

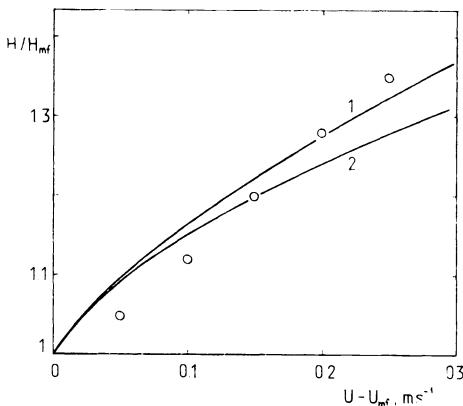


FIG. 3

Comparison of the bed expansion measured on the alumina spheres-air system with the predictions of different correlations. Experimental data points ( $\circ$ ) measured at  $410^\circ\text{C}$ ,  $D = 0.1$  m,  $\bar{d}_p = 0.81$  mm; Best and Yates<sup>18</sup>. The solid lines show the bed expansion ratio predicted by different correlations. 1 Babu et al.<sup>24</sup>; 2 Tamarin and Teplickii<sup>23</sup>

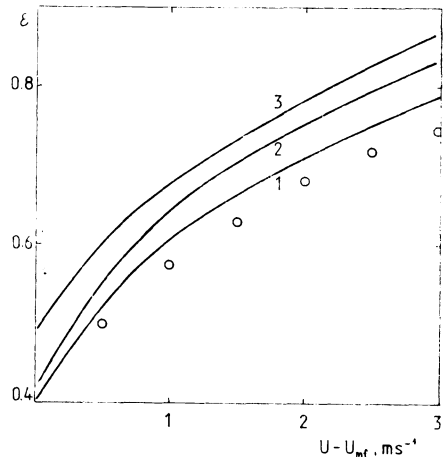


FIG. 4

Comparison of the bed voidage measured on the sand particle-air system with the predictions of different correlations. Experimental data points ( $\circ$ ),  $D = 0.11$  m,  $\bar{d}_p = 1.02$  and  $2.6$  mm, Denloye<sup>25</sup>. The solid lines show the bed voidage predicted by different correlations. 1 Hsiung and Thodos<sup>19</sup>, Eqs (12) and (13); 2 Hsiung and Thodos<sup>19</sup>, relationship normalized with respect to the free-fall conditions ( $Re_t, 1$ ); 3 Thonglimp et al.<sup>22</sup>

experimental data of the different authors which are presented in Figs 1–3 indicate that the beds with larger and lighter ( $\rho_s$ ) particles expand in smaller columns more than those with heavier and smaller solids in large vessels.

Predictions of the relationships of Hsiung and Thodos<sup>19</sup> and Thonglimp et al.<sup>22</sup> are confronted in Fig. 4 with the experimental data collected by Denloye<sup>25</sup>. As can be seen, there is a general agreement between the predictions and experiment. It is apparent that at the freely bubbling beds it is more practical to use Eq. (12) which is normalized with respect to the minimum fluidizing conditions.

Aside from the excess gas velocity the division of gas between the emulsion and bubble phase appears to be a major factor in the hydrodynamic models for expansion of the freely bubbling fluidized beds. Such models have to be employed when the local bubble gas hold-up is needed, as for example in the modelling of fluidized bed reactors. However, the empirical correlations should not be neglected, particularly, with respect to their ready use.

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#### SYMBOLS

$A$	$= (22 \cdot 27 / (U - U_{mf}))^2 \cdot D_{bm}$
$Ar$	$= d_p^3 g \rho_f (\rho_s - \rho_f) / \mu_f^2$ Archimedes number
$B$	$= (22 \cdot 27 / (U - U_{mf}))^2 \cdot (D_{bm} - D_{bo})$
$d_p$	particle diameter, m
$\bar{d}_p$	mean particle diameter, m
$d_b$	bubble diameter, m
$d_{vo}$	initial bubble diameter, m
$D$	diameter of the vessel, m
$D_{bm}$	maximum bubble diameter defined in ref. <sup>3</sup> , m
$D_{bo}$	initial bubble diameter defined in ref. <sup>3</sup> , m
$F$	cross-sectional area of the vessel, m <sup>2</sup>
$g$	acceleration due to gravity, m s <sup>-2</sup>
$h$	distance above the distributor plate in the vertical direction, m
$H$	average height of the expanded bed, m
$H_{mf}$	height of bed at the point of incipient fluidization, m
$k$	parameter given by Eq. (13)
$m$	parameter defined in ref. <sup>3</sup>
$n$	constant
$P$	pressure, N m <sup>-2</sup>
$Re$	$= U d_p \rho_f / \mu_f$ Reynolds number
$Re_{mf}$	$= U_{mf} d_p \rho_f / \mu_f$ Reynolds number at the minimum fluidization
$u_b$	mean absolute rising velocity of bubbles, m s <sup>-1</sup>
$U$	superficial gas velocity, m s <sup>-1</sup>
$U_{mf}$	Reynolds number at the minimum fluidization, m s <sup>-1</sup>
$W$	mass of particles, kg
$\delta$	parameter



$\varepsilon$	local bed voidage
$\bar{\varepsilon}$	mean bed voidage
$\varepsilon_b$	fraction of bed volume occupied by bubbles at the level $h$
$\bar{\varepsilon}_b$	overall fraction of bed volume occupied by bubbles
$\varepsilon_{mf}$	bed voidage at the point of minimum fluidization
$\mu_f$	fluid viscosity, $\text{kg (m s)}^{-1}$
$\rho_f$	fluid density, $\text{kg m}^{-3}$
$\rho_g$	gas density, $\text{kg m}^{-3}$
$\rho_s$	particle density, $\text{kg m}^{-3}$
$\psi$	ratio of the actual visible bubble flow rate to the gas flow rate given by $(U - U_{mf}) F$

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